Intellectual Journal of Energy Harvesting and Storage
Vol. 3, No. 2, December 2025, pp. 69~73
ISSN: 2963-6272, DOI: 10.11591/ehs.v3i2.pp69-73

Harvesting energy in nonlinear systems through time-delay mechanisms

Zakaria Ghouli[1,2]
[1]École Royale Navale (Royal Naval School), Casablanca, Morocco
[2]Polydisciplinary Faculty of Taroudant, University Ibn Zohr, Taroudant, Morocco

Article Info

Article history:

Received June 25, 2025
Revised Nov 3, 2025
Accepted Dec 14, 2025

Keywords:

Duffing–Van der Pol oscillator
Energy harvesting
Piezoelectric harvesting device
Quasi-periodic
Time-delay mechanisms

ABSTRACT

Energy harvesting (EH) from ambient vibrations is a promising approach for powering self-sustained devices. Nonlinear oscillators with time-delay feedback have attracted attention for their ability to broaden operating ranges and improve efficiency. Conventional linear harvesters and even nonlinear devices often suffer from instabilities, hysteresis, and limited bandwidth. Understanding how periodic and quasi-periodic (QP) vibrations contribute to EH in systems with dual time delays remains an open challenge. Prior studies have shown that time delay in mechanical subsystems can induce large-amplitude QP oscillations, while piezoelectric coupling with delay can enhance power output. However, most analyses treat mechanical and electrical delays separately. The combined influence of distinct delays in both mechanical and electrical components has not been systematically investigated, leaving unclear how dual-delay mechanisms affect stability and harvested power. This study models an EH system as a delayed Duffing–van der Pol oscillator coupled with a delayed piezoelectric circuit. Using perturbation methods and numerical simulations, we derive periodic and QP solutions near delay-induced parametric resonance and quantify harvested power. Results show that small mechanical delay amplitudes favor periodic vibrations, while larger amplitudes destabilize them, shifting energy extraction to QP vibrations with superior performance. Electrical delay further enhances harvested power across specific parameter ranges. These findings advance the theoretical foundation of nonlinear EH, highlighting QP vibrations as a viable strategy for efficient broadband energy extraction. The work provides design guidelines for delay-controlled harvesters and suggests future extensions to experimental validation and multi-degree-of-freedom systems.

Corresponding Author:

Zakaria Ghouli
École Royale Navale (Royal Naval School)
Casablanca, Morocco
Email: ghoulizakaria@gmail.com

1. INTRODUCTION

Energy harvesting (EH) systems are increasingly important for powering self-sustained devices in wireless sensing, biomedical implants, and smart infrastructure. Traditional linear harvesters often suffer from narrow bandwidth and limited efficiency. To overcome these shortcomings, nonlinear stiffness has been introduced into mechanical elements, improving performance in both monostable devices with hardening behavior [1]–[4] and bistable configurations [5]–[7].

Despite these advances, nonlinear attachments can be undermined by instabilities and jump phenomena near the edges of the stable branch in the frequency response [8]. Self-excited harvesters with linear stiffness can generate limit-cycle (LC) oscillations that enable energy extraction, but these oscillations may become unstable through secondary Hopf bifurcations, leading to quasi-periodic (QP) vibrations [9], [10]. In some contexts, such as aerodynamic or base excitations, QP vibrations reduce harvested power once flutter speed is exceeded [11], [12]. This highlights the challenge of maintaining stable and efficient EH across broad operating conditions.

Recent studies have shown that introducing time delay can fundamentally alter system dynamics. Time-delayed feedback has been demonstrated to produce large-amplitude QP oscillations over wide parameter ranges [13]. Investigations into delayed van der Pol-type harvesters with modulated delay amplitude confirmed that QP oscillations can contribute effectively to EH [14]. Scenarios with delay applied to both mechanical and electrical subsystems revealed that maximum power output does not always coincide with maximum system response amplitude [15]. Similarly, delayed Duffing oscillators coupled with piezoelectric circuits have been shown to support energy extraction over wide frequency ranges, avoiding hysteresis and instabilities near resonance [16]. Time delay has also been used to extend the dynamic range of nonlinear harvesters with damping [17], and modulation of delay amplitude has been shown to induce advantageous QP vibrations outside resonance regions [18].

While these studies highlight the potential of time-delay mechanisms, most focus on either mechanical delay or electrical delay in isolation. The combined influence of distinct delays in both subsystems has not been systematically analyzed. Moreover, although QP vibrations have been identified as beneficial in some cases, their stability and contribution to EH efficiency remain insufficiently characterized across different parameter regimes. This gap limits the practical design of harvesters that exploit dual-delay mechanisms.

Building on prior work, the present study develops and analyzes a system combining a delayed Duffing–van der Pol oscillator with a piezoelectric circuit that also incorporates delay. Using perturbation techniques and numerical simulations, we derive periodic and QP solutions near delay-induced parametric resonance, quantify harvested power, and establish stability charts. Particular attention is given to how time delays in the electrical subsystem influence harvesting efficiency, complementing naturally occurring mechanical delays observed in processes such as milling and turning [19]–[21].

By clarifying the interplay between mechanical and electrical delays, this work advances the theoretical foundation of nonlinear EH. The findings provide design guidelines for optimizing harvesters under broadband excitations, demonstrating that maximum power output does not necessarily align with maximum oscillation amplitude. More broadly, the study contributes to the development of efficient, delay-controlled harvesters for real-world applications, supporting the transition toward self-powered systems in engineering and industrial contexts.

2. FORMULATION OF THE ENERGY HARVESTER MODEL AND ANALYSIS

The EH system under investigation is based on a delayed Duffing–Van der Pol oscillator mechanically coupled to an electrical circuit via a piezoelectric transducer, as illustrated in the schematic of Figure 1. Both the mechanical and electrical subsystems are subject to delayed feedback. The dimensionless form of the system's governing equations is given (1) and (2):

$$\ddot{x}(t) + \delta\dot{x}(t) + \lambda\dot{x}(t)x(t)^2 + x(t) + \gamma x(t)^3 - \chi v(t) = \alpha\, x(t-\tau_1) \quad (1)$$

$$\dot{v}(t) + \beta v(t) + \kappa\dot{x}(t) = \alpha_3 v(t-\tau_2) \quad (2)$$

In this system, x(t) denotes the relative displacement of the rigid mass m, and v(t) represents the voltage across the resistive load. The parameter δ refers to the mechanical damping coefficient, while λ and β correspond to the reciprocal of the electrical time constant and the electrical damping, respectively. The term γ characterizes the stiffness of the mechanical structure. Piezoelectric coupling is represented by χ on the mechanical side and κ in the electrical circuit. Delayed feedback is introduced in both subsystems: the mechanical part is governed by a feedback gain α and a time delay τ_1, while the electrical subsystem includes a gain α_3 and a delay τ_2. It is worth noting that the delay in the mechanical branch is considered intrinsic to the dynamics of the harvesting system, as observed in applications such as milling and turning processes [19]–[21]. Conversely, the delay in the electrical loop is deliberately introduced to enhance the harvester's power output [15]. The control parameter α serves as a modulated delay gain associated with the displacement feedback.

$$\alpha = \alpha_1 + \alpha_2\cos(\omega t) \quad (3)$$

Here, α_1 denotes the constant (unmodulated) component of the delay amplitude, while α_2, ω represent the amplitude and frequency of the modulation, respectively. It should be noted that modulated delay amplitudes have been extensively utilized to enhance EH performance [14], [15], [18].

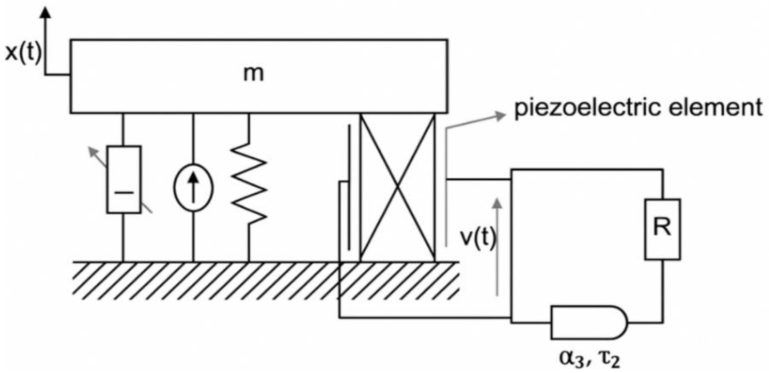


Figure 1. Schematic description of the EH system

It is important to highlight that the scenario in which nonlinear stiffness is absent ($\gamma = 0$) and the delays in the mechanical and electromagnetic components are equal has been examined in [15]. Additionally, the situation involving a linear damper and an unmodulated time delay was analyzed in [16]. In this study, we focus on the impact of the time delay in the electrical circuit on optimizing the EH performance of the harvester described by (1) and (2). We assume that the delays in the mechanical and electrical components differ not only in their timing but also in their amplitudes.

The system's response is analyzed in the vicinity of delay-induced parametric resonance by applying the resonance condition $1 = \frac{\omega^2}{4} + \sigma$, where σ is a detuning parameter. To carry out the analysis, the method of multiple scales [22] is employed. This approach leads to the derivation of the steady-state response corresponding to periodic solutions of (1) and (2), which is governed by a sixth-order algebraic equation in the amplitude a.

$$(S_1 a + S_2 a^3)^2 + (S_5 a + S_6 a^3)^2 = (S_3^2 + S_4^2)a^2 \tag{4}$$

An expression for the average power is obtained by integrating the dimensionless form of the instantaneous power $P(t) = \beta v(t)^2$ over the period of the delay modulation T. This is given by (5):

$$P_{av} = \frac{1}{T}\int_0^T \beta v^2 dt \tag{5}$$

where $T = \frac{4\pi}{\omega}$

Using the maximization procedure, one obtains the maximum power response as (6):

$$P_{max} = [\frac{\beta \kappa^2 \omega^2}{(2\beta - 2\alpha_3 \cos(\frac{\omega \tau_2}{2}))^2 + (\omega + 2\alpha_3 \sin(\frac{\omega \tau_2}{2}))^2}]a^2 \tag{6}$$

In (4) and (6) are used to examine the influence of different system parameters on the steady-state response and on the maximum output power of the harvester. In order to evaluate the QP response as well as the QP vibration-based EH, we approximate the QP response using the second-step perturbation method [23]. The approximate amplitude $a(t)$ of the QP response reads.

$$a(t) = \sqrt{\frac{R^2}{2} + \frac{R^2 \nu^2}{2(S_5 - S_4)^2} + [\frac{R^2}{2} - \frac{R^2 \nu^2}{2(S_5 - S_4)^2}]\cos(2\theta t)} \tag{7}$$

and the envelope of the QP modulation is delimited by a_{min} and a_{max} given by (8) and (9):

$$a_{min} = min\{\sqrt{\frac{R^2}{2} + \frac{R^2\nu^2}{2(S_5-S_4)^2} \pm [\frac{R^2}{2} - \frac{R^2\nu^2}{2(S_5-S_4)^2}]}\} \quad (8)$$

$$a_{max} = max\{\sqrt{\frac{R^2}{2} + \frac{R^2\nu^2}{2(S_5-S_4)^2} \pm [\frac{R^2}{2} - \frac{R^2\nu^2}{2(S_5-S_4)^2}]}\} \quad (9)$$

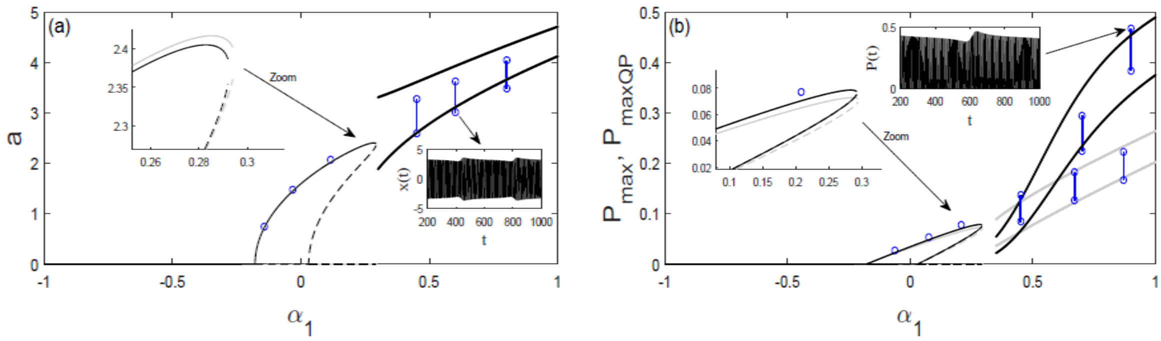
Consequently, the power and the maximum powers output in the QP modulation region are given, respectively, by (10) and (11):

$$P_{QP}(t) = \beta(\kappa e^{(\alpha_3 e^{\beta\tau_2}-\beta)t}\int_0^t \dot{x}(t')e^{(\beta-\alpha_3 e^{\beta\tau_2})t'}dt')^2 \quad (10)$$

$$P_{maxQP} = \frac{\beta\kappa^2\nu^2}{[(\beta-2\alpha_3\cos(\frac{\omega\tau_2}{2}))^2+(\nu+2\alpha_3\sin(\frac{\omega\tau_2}{2}))^2]}a^2 \quad (11)$$

where now a in (11) is derived from (8) and (9).

Figure 2 illustrates the relationship between the amplitude of the periodic and QP responses, along with the maximum output power amplitudes (P_{max}, P_{maxQP}), as a function of the unmodulated delay amplitude α_1. The results are presented for two cases: $\alpha_3 = 0$ (representing an undelayed circuit, shown by the grey line) and $\alpha_3 = \beta$ (indicating a delayed circuit, represented by the black line). The insets in the figure display the time histories of the amplitudes (Figure 2(a)) and the power responses (Figure 2(b)).



(a) (b)

Figure 2. Vibration and output power amplitudes versus the unmodulated delay amplitude α_1; (a) vibration amplitude time histories and (b) output power responses for delayed (black) and undelayed (grey) electric circuits

Figure 2 illustrates the relationship between the vibration and output power amplitudes and the unmodulated delay amplitude α_1. The results are shown for delayed ($\alpha_3=\beta$) and undelayed ($\alpha_3=0$) electric circuits, where black and grey lines denote the delayed and undelayed cases, respectively. Solid and dashed lines correspond to stable and unstable analytical predictions, while circles represent numerical simulations. The system parameters are set to $\alpha_2 = 0.25$, $\tau_1 = 5.2$, $\tau_2 = 4.2$, $\chi = 0.05$, $\beta = 0.05$, $\lambda = 0.2$, $\delta = -0.1$, $\gamma = 0.05$, $\omega = 2$, and $\kappa = 0.5$.

Figure 2 indicates that at small values of α_1, energy can only be harvested from periodic vibrations. However, as α_1 increases, the periodic solution becomes unstable, and energy extraction shifts exclusively to the QP vibrations, which demonstrate superior performance compared to the periodic output power. The plots also reveal that introducing a delay in the electrical circuit ($\alpha_3 \neq 0$, represented by the black line) results in a reduction of both the periodic and QP modulation amplitudes (Figure 2(a), black line), while the corresponding harvested power increases (Figure 2(b), black line). This suggests that the maximum power output does not necessarily align with the maximum amplitude of the oscillations.

3. CONCLUSION

In summary, we have investigated the EH performance of a delayed Duffing-van der Pol oscillator that is coupled with a delayed piezoelectric harvesting device. It is assumed that the mechanical and piezoelectric subsystems exhibit different time delays and varying delay amplitudes. The analysis focuses on the region near the delay parametric resonance, where the frequency of the delay modulation approaches twice the natural frequency of the oscillator. Perturbation techniques are employed to approximate both periodic and QP vibrations, which are utilized for EH. We explored the impact of the delay in the piezoelectric subsystem on the EH performance of the delayed Duffing-van der Pol harvester. Notably, it was demonstrated that the presence of modulated time delays in the mechanical subsystem leads to an optimal set of system parameters that maximizes both the amplitude of QP vibrations and the corresponding output power. To ensure the stability of the QP vibrations during the energy extraction process, a stability analysis was conducted, resulting in the establishment of a QP stability chart. The findings also indicate that the introduction of delay in the electrical circuit enhances the output power.

REFERENCES

[1] F. Peano, G. Coppa, C. Serazio, F. Peinetti, and A. D'angola, "Nonlinear oscillations in a MEMS energy scavenger," *Mathematical and Computer Modelling*, vol. 43, no. 11–12, pp. 1412–1423, Jun. 2006, doi: 10.1016/j.mcm.2005.07.007.

[2] B. P. Mann and N. D. Sims, "Energy harvesting from the nonlinear oscillations of magnetic levitation," *Journal of Sound and Vibration*, vol. 319, no. 1–2, pp. 515–530, Jan. 2009, doi: 10.1016/j.jsv.2008.06.011.

[3] R. Ramlan, M. J. Brennan, B. R. Mace, and I. Kovacic, "Potential benefits of a non-linear stiffness in an energy harvesting device," *Nonlinear Dynamics*, vol. 59, no. 4, pp. 545–558, Jul. 2009, doi: 10.1007/s11071-009-9561-5.

[4] S. C. Stanton, C. C. McGehee, and B. P. Mann, "Reversible hysteresis for broadband magnetopiezoelastic energy harvesting," *Applied Physics Letters*, vol. 95, no. 17, Oct. 2009, doi: 10.1063/1.3253710.

[5] M. F. Daqaq, "Transduction of a bistable inductive generator driven by white and exponentially correlated Gaussian noise," *Journal of Sound and Vibration*, vol. 330, no. 11, pp. 2554–2564, May 2011, doi: 10.1016/j.jsv.2010.12.005.

[6] A. Erturk, J. Hoffmann, and D. J. Inman, "A piezomagnetoelastic structure for broadband vibration energy harvesting," *Applied Physics Letters*, vol. 94, no. 25, Jun. 2009, doi: 10.1063/1.3159815.

[7] L. Van Blarigan, P. Danzl, and J. Moehlis, "A broadband vibrational energy harvester," *Applied Physics Letters*, vol. 100, no. 25, Jun. 2012, doi: 10.1063/1.4729875.

[8] D. D. Quinn, A. L. Triplett, A. F. Vakakis, and L. A. Bergman, "Energy Harvesting From Impulsive Loads Using Intentional Essential Nonlinearities," *Journal of Vibration and Acoustics*, vol. 133, no. 1, Dec. 2010, doi: 10.1115/1.4002787.

[9] K. Szabelski and J. Warminski, "Self-excited system vibrations with parametric and external excitations," *Journal of Sound and Vibration*, vol. 187, no. 4, pp. 595–607, Nov. 1995, doi: 10.1006/jsvi.1995.0547.

[10] M. Belhaq and A. Fahsi, "2:1 and 1:1 frequency-locking in fast excited van der Pol–Mathieu–Duffing oscillator," *Nonlinear Dynamics*, vol. 53, no. 1–2, pp. 139–152, Oct. 2007, doi: 10.1007/s11071-007-9302-6.

[11] A. Abdelkefi, A. H. Nayfeh, and M. R. Hajj, "Design of piezoaeroelastic energy harvesters," *Nonlinear Dynamics*, vol. 68, no. 4, pp. 519–530, Oct. 2011, doi: 10.1007/s11071-011-0233-x.

[12] A. Bibo and M. F. Daqaq, "Energy harvesting under combined aerodynamic and base excitations," *Journal of Sound and Vibration*, vol. 332, no. 20, pp. 5086–5102, Sep. 2013, doi: 10.1016/j.jsv.2013.04.009.

[13] M. Hamdi and M. Belhaq, "Quasi-periodic vibrations in a delayed van der Pol oscillator with time-periodic delay amplitude," *Journal of Vibration and Control*, vol. 24, no. 24, pp. 5726–5734, Aug. 2015, doi: 10.1177/1077546315597821.

[14] M. Belhaq and M. Hamdi, "Energy harvesting from quasi-periodic vibrations," *Nonlinear Dynamics*, vol. 86, no. 4, pp. 2193–2205, Feb. 2016, doi: 10.1007/s11071-016-2668-6.

[15] Z. Ghouli, M. Hamdi, and M. Belhaq, "Energy harvesting from quasi-periodic vibrations using electromagnetic coupling with delay," *Nonlinear Dynamics*, vol. 89, no. 3, pp. 1625–1636, May 2017, doi: 10.1007/s11071-017-3539-5.

[16] Z. Ghouli, M. Hamdi, F. Lakrad, and M. Belhaq, "Quasiperiodic energy harvesting in a forced and delayed Duffing harvester device," *Journal of Sound and Vibration*, vol. 407, pp. 271–285, Oct. 2017, doi: 10.1016/j.jsv.2017.07.005.

[17] A. S. Kammer and N. Olgac, "Delayed-feedback vibration absorbers to enhance energy harvesting," *Journal of Sound and Vibration*, vol. 363, pp. 54–67, Feb. 2016, doi: 10.1016/j.jsv.2015.10.030.

[18] Z. Ghouli, M. Hamdi, and M. Belhaq, "Energy Harvesting in a Duffing Oscillator with Modulated Delay Amplitude," in *IUTAM Symposium on Exploiting Nonlinear Dynamics for Engineering Systems*, Springer International Publishing, 2019, pp. 121–130. doi: 10.1007/978-3-030-23692-2_11.

[19] T. Kalmár-Nagy, G. Stépán, and F. C. Moon, "Subcritical Hopf Bifurcation in the Delay Equation Model for Machine Tool Vibrations," *Nonlinear Dynamics*, vol. 26, no. 2, pp. 121–142, Oct. 2001, doi: 10.1023/a:1012990608060.

[20] G. Stépán, R. Szalai, and T. Insperger, "Nonlinear Dynamics of High-Speed Milling Subjected to Regenerative Effect," *Nonlinear Dynamics of Production Systems*. Wiley, pp. 111–128, Jan. 2004. doi: 10.1002/3527602585.ch7.

[21] J. W. R Rusinek, A Weremczuk, "Regenerative model of cutting process with nonlinear Duffing oscillator," *Mechanics and Mechanical Engineering*, vol. 15, no. 4, pp. 131–145, 2011.

[22] D. T. M. A. H. Nayfeh, *Nonlinear Oscillations*. John Wiley & Sons, 2024.

[23] M. Belhaq and M. Houssni, "Quasi-Periodic Oscillations, Chaos and Suppression of Chaos in a Nonlinear Oscillator Driven by Parametric and External Excitations," *Nonlinear Dynamics*, vol. 18, no. 1, pp. 1–24, Jan. 1999, doi: 10.1023/a:1008315706651.