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Matrix inversion using multiple-input multiple-output adaptive filtering

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ABSTRACT

A new approach for matrix inversion is introduced. The approach is based on vector representation of multiple-input multiple-output (MIMO) channel matrix, in which the channel matrix is described by a linear combination of channel vectors weighted by their respective system inputs. The MIMO system output is then fed into a bank of adaptive filters, wherein the response of a given adaptive filter is iteratively minimized to match its output to the given system input. In doing so, adaptive filters equalize the impact of respective channel vectors on the MIMO channel output, while simultaneously orthogonalizing themselves from all other channel vectors, forming the channel matrix inverse. The method demonstrates satisfactory convergence and accuracy in Monte Carlo simulations conducted with varying signal-to-noise ratios (SNRs) and matrix conditioning scenarios. The suggested approach, by virtue of its adaptable characteristics, can also be employed for time-varying linear equation systems.

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1. INTRODUCTION

Adaptive filtering is an important branch of digital signal processing. Over the years, it has found applications in many areas of science and engineering, including mobile telecommunication systems [1]–[3], deep learning systems [4], [5], radar systems [6], [7], geophysics [8], [9], radiology [10], [11], image processing [12], speech applications [13]–[16], and power engineering [17]–[19]. This can be attributed to the advantages of adaptive filters in terms of real-time adaptation, noise rejection, robustness, convergence speed, and efficiency [20]. However, the application of adaptive filtering has been largely restricted to single-input single-output systems.

Multiple-input multiple-output (MIMO) systems arise in important areas of science and engineering, e.g., MIMO wireless communication systems [21], MIMO control systems [22], and MIMO acoustic systems [23]. MIMO systems can be represented by, or reduced to, a set of linear equations that describe the system response to multiple inputs. The system of linear equations needs to be solved in order to perform the necessary function. Adaptive filters can leverage their versatility to MIMO systems in solving such system of linear equations by computing the inverse of the MIMO channel matrix. This work provides a proof-of-concept by presenting a novel iterative method, based on a bank of adaptive filters, for matrix inversion in the context of MIMO wireless communication systems.

The method is based on the proposed vector representation of the MIMO channel matrix, wherein the channel matrix is described by a linear combination of channel vectors weighted by their respective system inputs. The system output can then be fed into a bank of adaptive filters, where the response of a given adaptive filter is iteratively minimized to match its output to the given system input. In doing so, a given adaptive filter equalizes the impact of the respective channel vector on the MIMO channel output, while simultaneously orthogonalizing itself from all other channel vectors. The collective response vectors of all adaptive filters form the channel matrix inverse.

The proposed method is novel. As a proof of concept, its efficacy is evaluated via Monte Carlo simulations for a range of signal-to-noise ratio (SNR) and matrix conditions. Simulation results show that the proposed method achieves good convergence and accuracy properties. Further, the method has an added advantage that, due to its adaptive ability, it can update the matrix inverse if the system drifts over time, for example, in non-stationary channel conditions [24].

2. THEORY

2.1. System model

Consider a MIMO system with dimensions $N_T \times N_R$, with N_T representing the number of transmit nodes and N_R the number of receive nodes, such that $N_T = N_R$. The input signal vector $\mathrm{x}[n]$ passes through the channel and is multiplied by a channel matrix H to generate the output signal vector $\mathrm{y}[n]$.

$$\mathrm{y}[n] = \mathrm{H}\mathrm{x}[n] + \mathrm{v}[n] \tag{1}$$

Where $\mathrm{x}[n] = [x_o(n) \quad x_1(n) \quad \ldots \quad x_{N-1}(n)]^T$, $\mathrm{y}[n] = [y_o(n) \quad y_1(n) \quad \ldots \quad y_{N-1}(n)]^T$, and $\mathrm{v}[n] = [v_o(n) \quad v_1(n) \quad \ldots \quad v_{N-1}(n)]^T$ are column vectors of dimension $N \times 1$, with $x_i(n)$ and $y_i(n)$ respectively indicating the signal transmitted on and received at the i-th node at the time instant n. $\mathrm{v}[n]$ represents the noise vector with statistical characterization $N(0, \sigma^2\mathrm{I})$ with I being the identity matrix. The channel matrix H is of dimensions of $N \times N$, and is described as (2):

$$\mathrm{H} = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0(N-1)} \\ h_{10} & h_{11} & \cdots & h_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{(N-1)0} & h_{(N-1)1} & \cdots & h_{(N-1)(N-1)} \end{bmatrix} \tag{2}$$

In (1) can be rearranged as (3):

$$\mathrm{y}[n] = \sum_{i=0}^{N-1} \begin{bmatrix} h_{0i} \\ h_{1i} \\ \vdots \\ h_{(N-1)i} \end{bmatrix} x_i(n) + \mathrm{v}[n] \tag{3}$$

with $i = 0,1, \ldots, N-1$. An estimate of $x_i(n)$ can be iteratively obtained from $\mathrm{y}[n]$ by using an adaptive filter (Figure 1). N such adaptive filter blocks can be used to obtain an estimate of $\mathrm{x}[n]$ from $\mathrm{y}[n]$, with filter weights of the blocks minimizing the impact of channel matrix H on $\mathrm{y}[n]$ by effectively acting as an inverse of H.

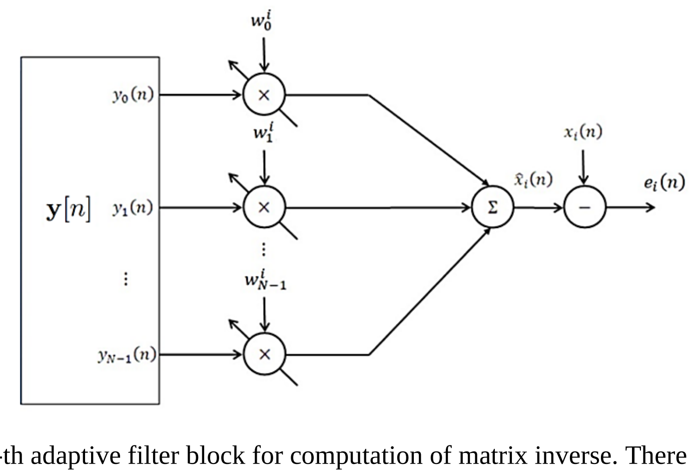

Figure 1. The i-th adaptive filter block for computation of matrix inverse. There are N such blocks. N represents the square matrix size

2.2. MIMO adaptive filter

For given value of i, in order to obtain an estimate $\hat{x}_i(n)$ of $x_i(n)$, an adaptive filter block (Figure 1) can be created with a weight vector $\mathrm{w}^i = \left[w_0^i, w_1^i, \dots, w_{N-1}^i\right]^T$, so that the i-th adaptive filter output $x_i(n)$ is:

$$\hat{x}_i(n) = \mathrm{w}^{i^T}\mathrm{y}[n] \quad (4)$$

Mean squared error (MSE) between the filter output $\hat{x}_i(n)$ and the desired output $x_i(n)$ is defined as (5) [25]:

$$E\{e_i^2(n)\} = \sigma_{x_i}^2 - 2\mathrm{w}^{i^T}\mathrm{p}_i + \mathrm{w}^{i^T}\mathrm{R}\mathrm{w}^i \quad (5)$$

with error $e_i(n) = x_i(n) - x_i(n)$. $\sigma_{x_i}^2 = E\{x_i^2(n)\}$ represents the power of the i-th desired signal $x_i(n)$. $N \times 1$ cross-correlation vector, p_i, between filter input and desire output is defined as (6):

$$\mathrm{p}_i = [p_{0i}(n) \quad p_{1i}(n) \quad \dots \quad p_{(N-1)i}\,(n)] \quad (6)$$

with $p_{ji}(n) = E\{y_j(n)x_i(n)\}$ and $j = 0,1,\dots,N-1$. $N \times N$ autocorrelation matrix R of the filter input $\mathrm{y}[n]$ is defined as (7):

$$\mathrm{R} = \begin{bmatrix} r_{00}(n) & r_{01}(n) & \dots & r_{0(N-1)}(n) \\ r_{10}(n) & r_{11}(n) & \dots & r_{1(N-1)}(n) \\ \vdots & \vdots & \ddots & \vdots \\ r_{(N-1)0}(n) & r_{(N-1)1}(n) & \dots & r_{(N-1)(N-1)}(n) \end{bmatrix} \quad (7)$$

such that $r_{ij}(n) = E\{y_i(n)y_j(n)\}$. Gradient of MSE taken with respect to the filter weights, ∇, leads to the Wiener-Hopf equation [25]:

$$\mathrm{w}_{\mathrm{opt}}^i = \mathrm{R}^{-1}\mathrm{p}_i \quad (8)$$

with ∇ being:

$$\nabla = -2\mathrm{p}_i + \mathrm{R}\mathrm{w}^i \quad (9)$$

For $i = 0,1,\dots,N-1$:

$$\mathrm{W}_{opt}^T = \mathrm{R}^{-1}\mathrm{P} \quad (10)$$

such that $\mathrm{W}_{\mathrm{opt}}^T = [\mathrm{w}_{\mathrm{opt}}^0, \mathrm{w}_{\mathrm{opt}}^1, \dots, \mathrm{w}_{\mathrm{opt}}^{N-1}]$ and $\mathrm{P}_{\mathrm{opt}} = [\mathrm{p}_0, \mathrm{p}_1, \dots, \mathrm{p}_{N-1}]$. W, which attempts to achieve a minimum MSE estimate of $\mathrm{x}[n]$ from $\mathrm{y}[n]$ by minimizing the impact of H on the latter, effectively acts as an inverse of H. Iterative computation of $\mathrm{W}_{\mathrm{opt}}$ is discussed in next section.

2.3. Iterative implementation

A starting choice for computing $\mathrm{W}_{\mathrm{opt}}$ iteratively can be the Steepest Descent algorithm [25] due to the simplicity of the form:

$$\mathrm{w}^i[n+1] = \mathrm{w}^i[n] - \mu\nabla \quad (11)$$

μ represents algorithm's iteration step-size. Substituting the value of the gradient from (9) in (11), and rearranging leads to:

$$\mathrm{w}^i[n+1] = \mathrm{w}^i[n] + 2\mu\mathrm{y}[n]e_i(n) \quad (12)$$

In (12) represents famous least mean squares (LMS) algorithm [25]. A concern in the implementation of LMS algorithm is the choice of μ. Large value of μ may lead to faster convergence but poor accuracy. Small value of μ may lead to higher accuracy but slower convergence. One way to overcome the issue is to make μ inversely proportional to the input signal energy, which in this case is $\mathrm{y}^T[n]\mathrm{y}[n]$. This leads to the normalized least mean squares (NLMS) algorithm [25]:

$$\mathrm{w}^i[n+1] = \mathrm{w}^i[n] + \frac{1}{\mathrm{y}^T[n]\mathrm{y}[n]}\mathrm{y}[n]e_i(n) \quad (13)$$

For $i = 0,1,\dots,N-1$,

$$\mathrm{W}[n+1] = \mathrm{W}[n] + \frac{1}{\mathrm{y}^T[n]\mathrm{y}[n]}\mathrm{y}[n]\mathrm{e}^T[n] \quad (14)$$

with $\mathrm{e}[n] = [e_0(n), e_1(n), \dots, e_{N-1}(n)]^T$.

3. METHODS

Simulations were performed for determining the efficacy of the proposed method. A 5×5 autocorrelation matrix of the first-order autoregressive process, AR(1), is used for simulating the MIMO channel matrix H. A 5×5 matrix results in five adaptive filter blocks, i.e., $i = 0, 1, \dots, 4$. Five type of simulations were performed. In the first stage, filter weights $\mathrm{w}^i[n]$ in (13), representing first-row estimates of W, were computed, overlaid and displayed as a function of the iterations performed by the NLMS algorithm as shown in Figure 2. In the second stage, square of the error term $\mathrm{e}[n]$ in (14), representing the individual error estimates between the respective desired outputs $x_i(n)$ and the estimated outputs $\hat{x}_i(n)$, were computed, overlaid and presented as function of NLMS iterations as presented in Figure 3. In the third stage, element-wise MSE between respective rows of $\mathrm{W}[n]$ in (14) and those of an exact matrix inverse were computed, overlaid and displayed as a function of number of NLMS iterations displayed in Figure 4. First, second, and third stage simulations were performed at SNR of 60 dB and one Monte Carlo noise run. In the fourth stage, element-wise MSE between the $\mathrm{W}[n]$ in (14) and the exact matrix inverse was computed and displayed as function of SNR ranging from 0 to 100 dB in the increments of unity, with $n = 110$ and 100 Monte Carlo runs for noise simulation as shown in Figure 5. First four simulations were carried out with $\alpha = 0.4$. In the fifth stage, four simulations were performed for α ranging from 0.2 to 0.8 in increments of 0.2; results were overlaid and displayed in Figure 6. All simulations were performed in MATLAB (Mathworks, Natick, MA, USA) using in-house written scripts.

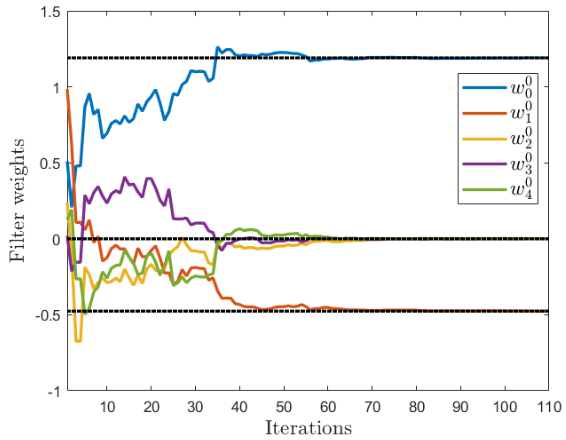

Figure 2. Filter weights $\mathrm{w}^i[n]$ in (13) with $i = 0$, depicting first-row estimates of inverse matrix W

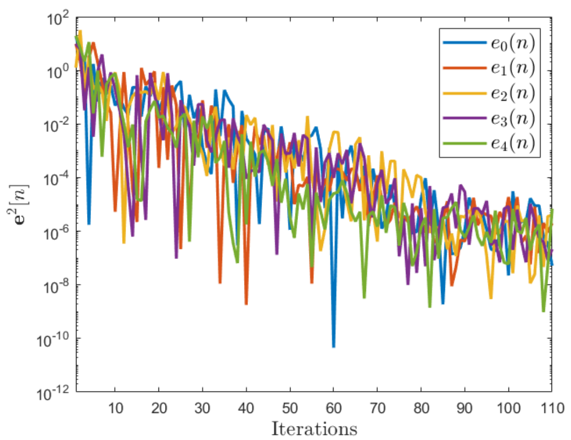

Figure 3. Squared NLMS-error term $\mathrm{e}[n]$ in (14) showing algorithm convergence properties

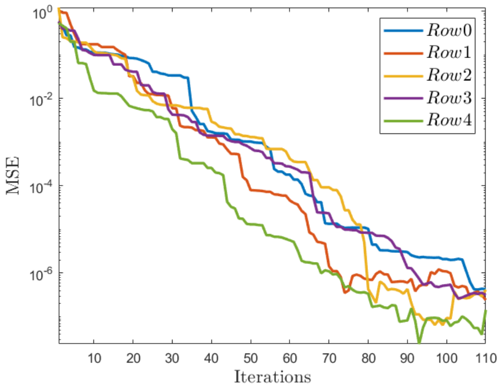

Figure 4. Element-wise MSE between respective rows of W and H^{-1}

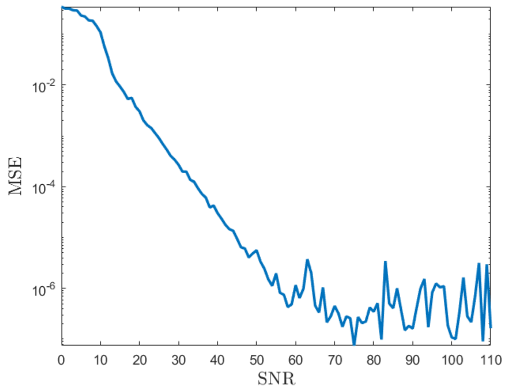

Figure 5. Element-wise MSE between W and H^{-1}, computed for a range of SNR values

4. RESULTS AND DISCUSSION

Figure 2 displays rapid convergence properties of representative estimates of the first adaptive filter block. Adaptive filter estimates converge to actual values in nearly less than seventy iterations. Also, once the estimates converge, they do not diverge or oscillate around the converged values. Figure 3 shows the row-wise squared NLMS-error, which demosntrates that the NLMS, employed as an optimization algorithm to compute the estimates, remains stable and does not diverge or oscillate. This can be attributed to the ability of the NLMS algorithm to adjust its step-size according to the input signal energy [25]. NLMS reduces the step-size to avoid the gradient noise, if the error is small; and if the error is large, NLMS increases the step-size to avoid convergence lag [25]. In addition, the instantaneous squared error reaches a value of 10^{-6} at the end of the eightieth iteration. This can be visualized in Figure 4, for all rows of the computed inverse matrix, wherein the element-wise MSE of the rows reaches the threshold of 10^{-6}, also at around the eightieth iteration. Figures 2 to 4 show the convergence and accuracy properties of the algorithm at SNR=60 dB, whereas Figure 5 shows the convergence and accuracy properties of the method for an SNR range of 0-110 dB. Up to 60 dB, the algorithm displays a negative log-linear trend between the MSE and the SNR, and the MSE enters a steady-state at nearly around 60 dB, without displaying any further reduction. This behavior becomes marked in Figure 6, where the results are overlaid for different condition numbers of H. For α=0.2, where the condition number of H is 2.01, MSE continues to decrease log-linearly with increase in SNR, reaching nearly 10^{-10} at 110 dB. On the other hand, for α=0.6 (condition number 9.74), MSE not only reaches a steady state comparatively early, that is, at 30 dB, but also achieves a higher steady state value of 10^{-2}; this trend continues for α=0.8, where the condition number increases to 29.76. At α=1, H becomes singular.

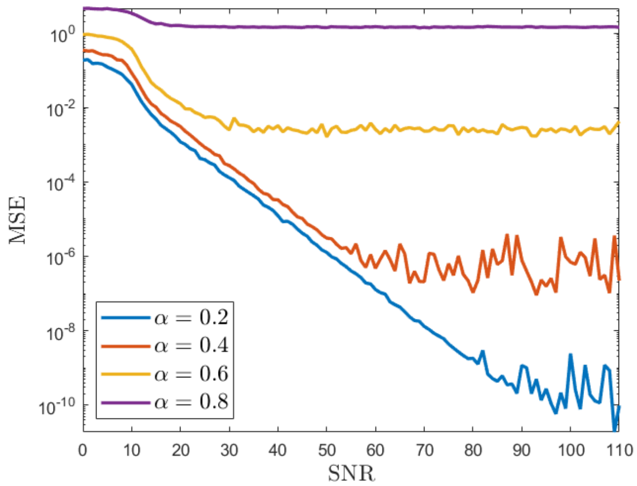

Figure 6. Element-wise MSE between W and H^{-1}, computed for range of matrix conditions

5. CONCLUSION

A novel matrix inversion method based on MIMO adaptive filtering is presented. Monte Carlo simulation results demonstrate that the method has good convergence and accuracy properties. The proposed method has the ability to iteratively adapt to changes in system conditions, which makes it suitable for practical implementation in systems with time-varying properties, e.g., MIMO wireless systems, MIMO acoustic systems, and MIMO control systems.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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Javed Iqbal	✓	✓				✓		✓		✓		✓		

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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