Computer Science and Information Technologies
Vol. 3, No. 2, July 2022, pp. 126~136
ISSN: 2722-3221, DOI: 10.11591/csit.v3i2.p126-136

Fuzzy formal concept analysis: approaches, applications and issues

Mohammed Alwersh, László Kovács
Department of Information Technology, József Hatvany Doctoral School for Computer Science and Engineering, University of Miskolc, Miskolc-Egyetenvaros, Hungary

Article Info

Article history:

Received Aug 28, 2021
Revised Jun 10, 2022
Accepted Jun 24, 2022

Keywords:

Formal concept analysis
Fuzzy FCA
Fuzzy logic

ABSTRACT

Formal concept analysis (FCA) is today regarded as a significant technique for knowledge extraction, representation, and analysis for applications in a variety of fields. Significant progress has been made in recent years to extend FCA theory to deal with uncertain and imperfect data. The computational complexity associated with the enormous number of formal concepts generated has been identified as an issue in various applications. In general, the generation of a concept lattice of sufficient complexity and size is one of the most fundamental challenges in FCA. The goal of this work is to provide an overview of research articles that assess and compare numerous fuzzy formal concept analysis techniques which have been suggested, as well as to explore the key techniques for reducing concept lattice size. as well as we'll present a review of research articles on using fuzzy formal concept analysis in ontology engineering, knowledge discovery in databases and data mining, and information retrieval.

Corresponding Author:

Mohammed Alwersh
Department of Information Technology, Faculty of Mechanical Engineering and Information Science
József Hatvany Doctoral School for Computer Science and Engineering, University of Miskolc
3515 Miskolc, Miskolc-Egyetenvaros, Hungary
Email: alwersh.mohammed.ali.daash@student.uni-miskolc.hu

1. INTRODUCTION

The mathematical foundation of formal concept analysis (FCA) is based on lattice theory [1]. The data analysis in FCA begins with a supplied cross table, wherein every row corresponds to the set of objects, every column refers to a collection of attributes, and the values of the cross-table indicates the relationship between them. The concept lattice is acknowledged as one of the primary outputs of formal concept analysis, reflecting generalization and specialization between the cross table's created formal concepts [2]. Formal concept is a fundamental unit of concept that play an essential role in knowledge processing containing the extent part (sets of objects) and the intents part (corresponding common attributes). FCA's traditional setting considers the context to be a table., where the rows in the context representing the domain objects, and the columns of the context refer to the attributes for each object in the domain under the study. The cross-table (context) inputs carrying the values ones and zeros (X symbol\ empty) based on whether or not an object possesses the attribute. As a result, the basic formal concept analysis is superior for the attributes that has a crisp value (0s or 1s). At the same time, features might be vague rather than precise(crisp). FCA has successfully been enhanced with a fuzzy setting to accommodate the ambiguity and vagueness in data. For example, if we ask if a man with a height of 170 cm is tall, we'll probably get an answer like "not totally tall but almost tall" or "to a high degree tall,". Lotfi Aliasker Zadeh introduced such a notion in fuzzy logic [3] to assign a truth level of belonging to an object based on the fuzzy attributes that the object contains. An L-scale of truth degrees (degree of belonging) is used to compute the degrees of belongings. One of the most popular

L-scale selections is the interval [0, 1]. To return to our earlier example, a guy of (170 cm) in height is tall to a level of 0.7. As a result, rather than 0 or 1, as in traditional FCA, the context values are becoming degrees from the interval [0,1].

FCA's immense potential is promoted by the concept lattice generated by the collection of formal concepts which retrieved from the formal context of a domain under the study. In reality, the lattice is used in the vast majority of applications., which is often depicted by a line diagram (Hass diagram). Large numbers of formal concepts might perhaps be produced from a tiny quantity of data [4]. In reality, FCA may lead to a great deal of computational complexity, and even with a small dataset, the resulting concept lattice can be huge [5]. For many applications, the computational cost is still too high. Furthermore, examining the final lattice might be challenging due to the enormous number of formal concepts and the intricacy of concept interactions [6]. In fact, obtaining a concept lattice of adequate complexity and scale is one of the most critical obstacles of employing formal concept analysis [4].

Concept lattice reduction strategies come in many different forms., each with its own set of advantages and disadvantages. Some of them purge the concept lattice from redundant information. Generally, their main goal is to seek the smallest number of objects or attributes which preserve the hierarchy order of the original concept lattice [7], [8]. Other techniques aim to create an abstraction of the concept lattice, or to achieve a high level of simplification that reveals the genuinely important aspects [9]. Lastly, some types of techniques employ a relevance criterion to select formal concepts, objects, or attributes [10]. The primary goal of this work is to clarify the relationship between the various approaches to Fuzzy formal concept analysis and to discuss the main issues associated with using it. The second direction in this work is to present a review of research articles on using fuzzy formal concept analysis in various applications such as knowledge discovery in databases and data mining, information retrieval, and ontology engineering.

The remainder of this paper is composed: In section 2 will give a theoretical background to the fuzzy formal concept analysis. In section 3, we will go over and compare the most important approaches for fuzzy formal concept analysis that have been proposed. Section 4 will provide a quick overview of the use of fuzzy formal concept analysis in several fields. In section 5, we will discuss the main techniques to reduce the size of concept lattice that considered as a main issue in several applications that used FCA as an analytical method.

2. FUZZY FORMAL CONCEPT ANALYSIS

Formal concept analysis is a theoretical framework that provides a foundation for conceptual data analysis and knowledge processing. It allows the representation of the relationships between objects and attributes in a specific domain [11]. Formal concept analysis offers a different graphical representation of tabular data that is easier to navigate and use [1]. A more detailed overview is provided in [1].

2.1. Formal concept analysis (FCA)

The ideas of a formal context are taken into account by FC), which describes the attributes of every object from the domain. Accordingly, a formal context may be conceptualized as a binary connection between both the object group and the attribute group, with values of 0 and 1. A cross-table (formal context) is given to FCA at the beginning, and it is described as a triple $K = (G, M, I)$, where G denotes a group of objects, M denotes a set of attributes, and I denotes a binary connection ($I \subseteq G \times M$). $(g, m) \in \mathrm{I}$, can be read as an object g has the attribute m.

The usage of the words "object" and "attribute" is instructive because it may be advantageous in many situations to select things that resemble other objects as formal objects and then select their properties as formal attributes. In the field of information retrieval, documents, for instance, might be thought of as being object-like and phrases, as being attribute-like. As seen in Figure 1(a) from [11], the context is frequently represented as a cross table, with rows denoting formal objects, columns denoting formal attributes, and crosses denoting interactions between them.

Definition 1. Formal Concept, a context $K = (G, M, I)$, for $A \subseteq G$ and for $B \subseteq M$ applying a derivation operator:

$$\mathrm{A}' = \{\mathrm{m} \in \mathrm{M} \mid \mathrm{gIm} \text{ for } \forall \mathrm{g} \in \mathrm{A}\} \tag{1}$$

$$\mathrm{B}' = \{\mathrm{g} \in \mathrm{G} \mid \mathrm{gIm} \text{ for } \forall \mathrm{m} \in \mathrm{B}\} \tag{2}$$

The set of all objects having all of the attributes from B is called A', whereas the set of all attributes shared by all objects from A is called B'. As a result, a formal concept is defined as a pair (A, B) for a formal context $(\mathrm{G}, \mathrm{M}, \mathrm{I})$, where $\mathrm{A} = \mathrm{B}'$ and $\mathrm{B} = \mathrm{A}'$ are satisfied. For a formal concept, A is referred to as the concept's extent and B as its intent.

Definition 2. For two concepts $\mathrm{c}_1 = (\mathrm{A}_1, \mathrm{B}_1)$, and $\mathrm{c}_2 = (\mathrm{A}_2, \mathrm{B}_2)$, c_1 is considered as a subconcept of c_2 (equivalanty c_2 is a superconcept of c_1), $(\mathrm{A}_1, \mathrm{B}_1) \leq (\mathrm{A}_2, \mathrm{B}_2) \Leftrightarrow \mathrm{A}_1 \subseteq \mathrm{A}_2$(or equivalently $\mathrm{B}_1 \subseteq \mathrm{B}_2$) The set of all formal concepts ordered in that way, indicates a complete lattice.

The Figure 1(a) depicts the line diagram for the concept lattice which constructed from the formal context that shows in the upper left corner of Figure 1(a). The concept lattice could be made up by the set of formal concepts which constructed from the formal context and the subconcept-superconcept relation between them [1]. The nodes in the line diagram refer to the formal concepts where the lower nodes noted in the diagram represented the formal objects and formal attributes are dipcted in the higher level of the diagram.

To obtain the extent of a formal concept, one must follow the descending path from the node to get the formal objects. In the Figure 1(a) we can notice that the formal objects for the node c_2 are (URL_3, URL_2, URL_5). To obtain the intent of a formal concept, one must follow the ascending path from the node to get the formal attributes like the formal attribute "study" located at the top of the node c_2. Note that c_2 is a formal concept with extent (URL_3, URL_2, URL_5) and intent (series, study). c_2 is consider as a subconcept of c_1.

2.2. Fuzzy formal concept analysis

FCA has recently been used in several applications where the domain representation contains uncertain and ambiguous information. A generalized Wille's model was one of the first studies to incorporate fuzziness into FCA [12]. Specifically, the use of a residuated lattice [13]–[16] to extend the original formal concept analysis by determining the truth degree for the assertions "object x has attribute y" in fuzzy formal contexts. Degrees are calculated using an L-scale of truth degrees. Normally, real values in the range [0, 1] are used to value L. As a result, instead of values from 0 or 1 as in the basic setting of classical FCA, the entries of the cross-table describing objects and attributes become degrees from L. This is known as a fuzzy formal concept analysis.

Definition 3. A fuzzy formal context is a triple $K = (G, M, I = \varphi(G \times M))$, G is the set of objects where M is the set of attributes and I represent a fuzzy set on $G \times M$. Every pair $(g, m) \in I$ has a membership value $\mu_I(g, m)$ taken from the interval [0,1]. The set $I = \varphi(G \times M) = \{((g, m), \mu_I(g, m)) \mid \forall g \in G, m \in M, \mu_I: G \times M \rightarrow [0,1]\}$ is a fuzzy relation $G \times M$.

Definition 4. A fuzzy set $\Phi(g)$ (fuzzy representation of g) can represent for every object g in a fuzzy formal context K as $\Phi(g) = \{(m_1, \mu_I(m_1)), (m_2, \mu_I(m_2)), \dots, (m_i, \mu_I(m_i))\}$ where $\{m_1, m_2, .., m_i\}$is refer to the set of attributes in the formal context K, $\mu_I(m_i)$ is refer to the membership related to the attribute m_i. Figure 1(b) depicts a fuzzy model of the formal context using a cross-table.

Definition 5. Let $K = (G, M, I)$ be a fuzzy formal context with a confidence threshold T, for $A \subseteq G$ we can define $A^* = \{m \in M \mid \forall g \in A: \mu_I(g, m) \geq T\}$, and for $B \subseteq M$ we can define $B^* = \{g \in G \mid \forall m \in B: \mu_I(g, m) \geq T\}$. A fuzzy formal concept of a fuzzy formal context with a threshold T, can be define as a pair $(\varphi(A), B)$, where $A \subseteq G$ and $\varphi(A) = \{g, \mu_{\varphi(A)}(g) \mid \forall g \in A\}$, $B \subseteq M$, $A^* = B$ and $B^* = A$, where every object g has a membership $\mu_{\varphi(A)}(g)$ defined as $\mu_{\varphi(A)}(g) = min_{m \in B}\, \mu_I(g, m)$. In the concept $(\varphi(A), B)$, A and B are the extent and the intent of the concept respectively.

The fuzzy formal context in Figure 1(b) has a confidence threshold T = 0.6. Where all objects-attributes relationships with membership values lower than 0.6 are hidden.

Definition 6. Given two fuzzy formal concepts like $(\varphi(A_1), B_1)$ and $(\varphi(A_2), B_2)$ of a fuzzy formal context (G, M, I). $(\varphi(A_1), B_1)$ is the subconcept of $(\varphi(A_2), B_2)$ denoted as $(\varphi(A_1), B_1) \leq (\varphi(A_2), B_2)$ if and inly if $\varphi(A_1) \subseteq \varphi(A_2)$ $(equivelanty\ B_2 \subseteq B_1)$.

For example, in the Figure 1(b) the concept c_5 is a subconcept of the concepts $c_2\ and\ c_3$, on the other hand the concepts $c_2\ and\ c_3$ are the superconcepts of the concept c_5.

Definition 7. Let $K = (G, M, I)$ be a fuzzy formal context, within K and a confidence threshold T, we can define a fuzzy concept lattice as a set of all fuzzy formal concepts of K partially order $\leq$ with confidence threshold T.

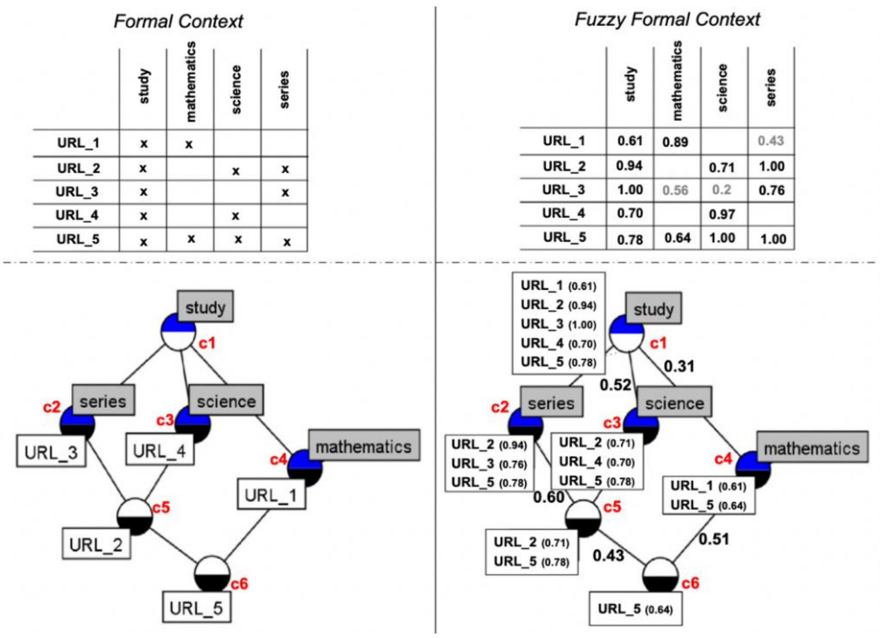
Definition 8: Given two formal concepts C_1, C_2, where $C_1 = (\varphi(A_1), B_1)$ is a superconcept of C_2 and $C_2 = (\varphi(A_2), B_2)$ is a subconcept of C_1, the similarity between C_1, C_2 is described as:

$$sim(C_1, C_2) = |\varphi(A_1) \cap \varphi(A_2)| \,/\, |\varphi(A_1) \cup \varphi(A_2)|$$

The operators ∩,∪ indicate the intersection and union (respectively) operations on a fuzzy set. T-norm and t-conorm are used to compute the fuzzy intersection and union. The minimum is the most widely used t-norm, whereas the maximum is the most widely used t-conorm. Assume two fuzzy sets $A\ and\ B$ with membership functions $\mu_A(x)$, $\mu_B(x)$, where $x \in U$(universe of discourse), the intersection and union operators are defined as $\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$ and $\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))$. As an illustration, consider the similarity of the fuzzy formal concept that calculated between the concepts $C_2 = \{(URL_2, URL_3, URL_5), (series, study)\}$ and $C_5 = \{(URL\_2, URL\_5), (science, study, series)\}$. That given in the Figure 1(b).

$$sim(C_2, C_5) = \frac{(|(min\ \{0.71,0.94\}) + (min\ \{0.78,0.78\})|)}{(|(max\ \{0.71+0.94\}) + (max\ \{0.78,0.78\}) + (max\ (0.76))|)} = 0.60$$

Figure 1 illustrates how fuzzy formal concept analysis (FFCA) and formal concept analysis (FCA) are modeled differently. The cross-table in the Figure 1(a) used in the classical FCA comprises binary values that describe the existence or absence of the link between objects and attributes. A cell with a value in the range of [0, 1] in a fuzzy setting, such as the cross-table in Figure 1(b), shows whether or not there is a link and offers an assessment of the strength of that association [11].



(a) (b)

Figure 1. The differences in the modeling of the classical FCA and Fuzzy FCA in (a) Formal concept analysis, and (b) Fuzzy formal concept analysis [11]

3. COMPARISON OF FCA APPROACHES

3.1. L-fuzzy concept lattice

The authors in their paper [16] were the first to indicate that FCA can be expanded to include fuzzy concepts. They proceed in the following manner: Let $L = (L, \leq,', \oplus, 0, 1)$ be a structure such that $(L, \leq,', 0, 1)$ indicates a complete lattice constrained by (0 and 1),′ is refer to unary complementation operation, where ⊕ be a t-conorm on L (a binary operation with the neutral element 0 that is associative, commutative, and associative).

Given an L-context (X, Y, I), define mappings $\uparrow : L^X \rightarrow L^Y$ and $\downarrow : L^Y \rightarrow L^X$:

$$A^{\uparrow}(y) = \wedge_{x\in X}(A(x)' \oplus I(x,y)), \tag{3}$$

$$B^{\downarrow}(x) = \wedge_{y\in Y}(B(y)' \oplus I(x,y)), \tag{4}$$

For $A \in L^X$ and $B \in L^Y$, put $\mathcal{B}(X,Y,I) = \{\langle \mathrm{A}, \mathrm{B}\rangle \in L^X \times L^Y | \mathrm{A}^{\uparrow} = \mathrm{B},\ \mathrm{B}^{\downarrow} = \mathrm{A}\}$ as well as describe a partial order $\leq$ on $\mathcal{B}(X,Y,I)$ by $\langle A_1, B_1\rangle \leq \langle A_2, B_2\rangle$ if and only if $A_1 \subseteq A_2$ (equivalently $B_2 \subseteq B_1$). To clarify, $'$ being a classical negation $(1' = 0\ and\ 0' = 1)$ and $\oplus$ is a classical disjunction $(a \oplus \mathrm{b} = \max(\mathrm{a}, \mathrm{b}))$. In their study [16], the authors establish some of the fundamental characteristics of $\uparrow$ and $\downarrow$. and show that $\mathcal{B}(X,Y,I)$ outfitted with $\leq$ is a complete lattice. The authors then expanded their strategy to incorporate what are known as implication operators [17]. Also noteworthy is that the authors addressed other FCA extensions in their context; see [18].

3.2. FCA and related structures in a fuzzy setting

Belohlavek [19] and Pollandt [20] independently proposed the basic concepts of FCA in fuzzy environment. They developed the following approach, which proved to be a viable method for developing FCA and related structures in a fuzzy environment. Let (X,Y,I) be an L-context, i.e., $I: X \times Y \to L$. For a fuzzy set $A \in L^X$ and $B \in L^Y$, suppose fuzzy sets $A^{\uparrow} \in L^Y$ and $B^{\downarrow} = L^X$ described by

$$A^{\uparrow}(y) = \bigwedge_{x \in X}(A(x) \to I(x,y)), \tag{5}$$

$$B^{\downarrow}(x) = \bigwedge_{y \in Y}(B(y) \to I(x,y)). \tag{6}$$

Using fundamental predicate fuzzy logic rules [15], [21], it is straightforward to understand that $A^{\uparrow}(y)$ is the truth degree of y is common for all objects in A" and $B^{\downarrow}(x)$ is the truth degree of x possesses all attributes from B." As a result, we may claim that (5) and (6) are exact generalizations of (1) and (2). Putting $\mathcal{B}(X,Y,I) = \{\langle \mathrm{A}, \mathrm{B}\rangle \mid A^{\uparrow} = B, B^{\downarrow} = A\}$. is refer to the set of all formal concepts $\langle \mathrm{A}, \mathrm{B}\rangle$, such that A is refer to the set of all objects that share all the attributes of B(the intent), and B is refer to the set of all attributes that shared by all objects of A(the extent). $\mathcal{B}(X,Y,I)$ refers to a collection of all formal concepts, where A denotes the collection of objects with all of B's features. known as the intent part, and B is the collection of all features that all A's objects share. known as the extent part. $\mathcal{B}(X,Y,I)$ is regarded as as a fuzzy concept lattice of the formal context (X,Y,I). The extent part of a formal concept and the intent of a formal concept (A,B) are both in general fuzzy sets, as in the method of Burusco and Fuentes-Gonzalez [16]. This represents the reality that concepts, in general, apply to objects and attributes to varying degrees, rather than simply 0 and 1. Putting

$$\langle \mathrm{A}_1, \mathrm{B}_1\rangle \leq \langle \mathrm{A}_2, \mathrm{B}_2\rangle \text{ iff } A_1 \subseteq A_2 \text{ (iff } B_2 \subseteq B_1) \tag{7}$$

For $\langle \mathrm{A}_1, \mathrm{B}_1\rangle, \langle \mathrm{A}_2, \mathrm{B}_2\rangle \in \mathcal{B}(X,Y,I), \leq$ represented the subconcept-superconcept hierarchy in $\mathcal{B}(X,Y,I)$

3.3. Fuzzy concept lattice with non-commutative conjunction

The authors [22], defined the fuzzy concept lattice $(L, \vee, \wedge, \otimes, \to, \Rightarrow, 0, 1)$ in their method, which combines fuzzy logic with a non-commutative conjunction $\otimes$ rather than a commutative conjunction. They claim that removing the commutativity condition is necessary in instances when the order of the terms of the conjunction concerns, in order to make the theory suitable for representing temporal data. In this case, the Galois connection will be made up of two pairs of functions, $\uparrow, \Uparrow: L^X \to L^Y$ and $\downarrow, \Downarrow: L^Y \to L^X$, each in a symmetric position to his partner. The authors further demonstrate that any non-commutative fuzzy logic concept lattice may be understood using their framework of extended concept lattices with non-commutative conjunction.

3.4. One-sided fuzzy concept lattice

In [23] and [24] separately devised the "One-sided fuzzy concept lattice" approach. The definitions of the authors get identical outcomes for $(\mathrm{X}, \mathrm{Y}, \mathrm{I}^{-1})$, $\mathrm{I}^{-1} \in \mathrm{L}^{\mathrm{X} \times \mathrm{Y}}$ defined by $\mathrm{I}^{-1}(\mathrm{x}, \mathrm{y}) = \mathrm{I}(\mathrm{x}, \mathrm{y})$, implying that the techniques are equivalent in terms of the function of objects and attributes. $\mathrm{L} = [0,1]$ is also used by the authors. The authors established two mapping operators for a fuzzy formal context (L-context), (a) f: $2^{\mathrm{X}} \to \mathrm{L}^{\mathrm{Y}}$ by $\mathrm{f}(\mathrm{A})(\mathrm{y}) = \bigwedge_{\mathrm{x} \in \mathrm{A}} \mathrm{I}(\mathrm{x}, \mathrm{y})$, where $\mathrm{A} \sqsubseteq \mathrm{X}$ (objects set), $\mathrm{f}(\mathrm{A}) \in \mathrm{L}$ (attributes fuzzy set). And (b) h: $\mathrm{L}^{\mathrm{Y}} \to 2^{\mathrm{X}}$ by $\mathrm{h}(\mathrm{B}) = \{\mathrm{x} \in \mathrm{X} \mid \text{each } \mathrm{y} \in \mathrm{Y}: \mathrm{B}(\mathrm{y}) \leq \mathrm{I}(\mathrm{x}, \mathrm{y})\}$ for each yY:B(y)I(x,y), where $\mathrm{B} \in \mathrm{L}^{\mathrm{Y}}$(attributes fuzzy set) and $\mathrm{h}(\mathrm{B}) \in \mathrm{X}$(objects set). Then, the authors put.

$$\mathcal{B}_{\mathrm{f,h}}(\mathrm{X}, \mathrm{Y}, \mathrm{I}) = \{\langle \mathrm{A}, \mathrm{B}\rangle \in 2^{\mathrm{X}} \times \mathrm{L}^{\mathrm{Y}} \mid \mathrm{f}(\mathrm{A}) = \mathrm{B}, \mathrm{h}(\mathrm{B}) = \mathrm{A}\}$$

The authors established that $\mathcal{B}_{\mathrm{f,h}}(\mathrm{X}, \mathrm{Y}, \mathrm{I})$ could be fitted by the partial order $\leq$, as specified in (7), to form a complete lattice; the researchers have attributed for such a method as a "one-sided fuzzy concept lattice". Be aware that concepts from $\mathcal{B}_{f,h}(X,Y,I)$ have crisp sets for their intentions and fuzzy sets for their extents.

3.5. Crisply generated fuzzy concepts

Regarding the problem of a possibly big set of formal concepts , the authors in their work [25] recommended just using a portion of $\mathcal{B}(X,Y,I)$, called $\mathcal{B}_c(X,Y,I)$, as opposed to using the complete $\mathcal{B}(X,Y,I)$. $\langle A,B\rangle \in \mathcal{B}(X,Y,I)$ can be called crispy if there is a subset exist $B_c \subseteq Y$ (of attributes), such that $A = B_c^{\downarrow}$ (thus, $B = B_c^{\downarrow\uparrow}$). Then, the complete lattice of crispy generated fuzzy concept represented by $\mathcal{B}_c(X,Y,I) = \{\langle A,B\rangle \in \mathcal{B}(X,Y,I) \mid \text{exists } B_c \subseteq Y: A = B_c^{\downarrow}\}$.

Consider the ways in which intentions are crisp sets and extrentions are fuzzy sets. in Yahia and Krajci's "One-sided fuzzy concept lattice" approach, whereas both extention part and intention part are generally fuzzy sets in the "Crisply Generated Fuzzy Concepts" approach. B (f,h) (X,Y,I) equipped with the partial order (≤) in the "One-sided fuzzy concept lattice" given in (7) is a complete lattice that is isomorphic to $\mathcal{B}_c(X,Y,I)$ equipped with the partial order acquired from $\mathcal{B}(X,Y,I)$. Additionally, there is an isomorphism for the equivalent notions, $\langle A,B\rangle \in \mathcal{B}_{f,h}(X,Y,I)$ and $(C,D) \in \mathcal{B}_c(X,Y,I)$ such that $A = C$, $B = D^{\downarrow\uparrow}$.

3.6. Generalized concept lattice

A "generalized concept lattice" is the objective of the author's investigation in [26]. In general, the author suggests that three sets of truth degrees (level of belonging) be taken into consideration: L_X (refer to the objects set), L_Y (attributes set), and L indicates the table entries (degree of attribute possession of objects). Assuming that X is objects set and Y is attributes set, the context that considered as a fuzzy context may be thought of as a triple (X,Y,I), where I denotes the L-relation between the objects set and the attributes set, i.e., $I \in L^{X\times Y}$. The author also asserts that L is a partly ordered set and that L_X and L_Y are complete lattices. ≤ is used to represent all partial orders on (L_X, L_Y, and L). In order to define arrow-operators, the author makes the following assumption: It is satisfied by: $\otimes: L_X \times L_Y \rightarrow L$.

$$a_1 \leq a_2 \Rightarrow a_1 \otimes b < a_2 \otimes b, \tag{8}$$

$$b_1 \leq b_2 \Rightarrow a \otimes b_1 < a \otimes b_2, \tag{9}$$

$$\text{If } a_j \otimes b \leq c \text{ for each } j \in J \text{ then } \left(\bigvee_{j\in J} a_j\right) \otimes b \leq c, \tag{10}$$

$$\text{If } a \otimes b_j \leq c \text{ for each } j \in J \text{ then } a \otimes \left(\bigvee_{j\in J} b_j\right) \leq c, \tag{11}$$

This is for each index set J and for all $a, a_j \in L_X, b, b_j \in L_Y$ and $c \in L$. To put it another way, there are three levels of truth $(L_1, L_2, L, \otimes, \leq, \ldots)$. If it meets (8)–(11), this structure is referred to be Krajci's structure.

Then, Krajci moves on to mappings the arrow-operations $\uparrow: L_X^X \rightarrow L_Y^Y$ and $\downarrow: L_Y^Y \rightarrow L_X^X$ by

$$A^{\uparrow}(y) = \bigvee\{b \in L_Y \mid \forall x \in X: A(x) \otimes b \leq I(x,y)\} \tag{12}$$

$$B^{\downarrow}(x) = \bigvee\{a \in L_X \mid \forall y \in Y: a \otimes B(y) \leq I(x,y)\} \tag{13}$$

The formal concepts in (X,Y,I) are defined as pairs $(A,B) \in L_X^X \times L_Y^Y$ fulfilling $A^{\uparrow} = B, B^{\downarrow} = A$.

$\mathcal{B} = \{\langle A,B\rangle \mid A^{\uparrow} = B,\ B^{\downarrow} = A\}$ (formal concepts) fitted with the partial order (≤) is a complete lattice (i.e., the generalized concept lattice for $\langle X,Y,I,\otimes\rangle$). In [27] establishes a fundamental theorem for an extended concept lattice.

4. APPLICATION DOMAINS

In numerous disciplines, formal concept analysis has been utilized in conjunction with fuzzy logic. In order to detect connections between demographic data and physical activity levels, Data from epidemiological surveys on physical exercise were examined using FCA by the authors of [28]. Later, Belohlavek *et al*. (2007, 2011) build on the work of Sklenar *et al*. (2005) and Sigmund *et al*. (2005) by aggregating Participants and using fuzzy values to express the relative strength of characteristics in the aggregated items. Based on biological characteristics analysis, the authors provide a framework for identifying ecological properties of organisms in [29]. The complicated structure of the dataset is formalized as a fuzzy many-valued context, which is then translated to a binary context using histogram scaling. The framework of the technique was based on the production and evaluation of formal concepts. The concepts were analyzed by a hydrobiologist, resulting in a collection of ecological features that were added to the initial context.

By fusing FCA and fuzzy characteristics, the authors of in [30] presented a framework that aids users in their discovery of semantic web resources. Lower and higher levels make up this structure. In the lowest layer, fuzzy multisets are created from the semantic representations of web services. The service's capabilities are represented in this representation (an OWL-S document). This representation, which is an OWL-S document, demonstrates the functionality of the service. Fuzzy C-Means clustering is used to group the web services into fuzzy clusters. Services that are near matches to the input request have been found using fuzzy matching. Using a fuzzy formal context, archetypes and ascribed ontological conceptions that are present or absent have been defined at the upper layer.

Ontology engineering is another research area that focuses on the relationships between individuals and classes. In [31], the authors use FCA in combined with fuzzy logic to automatically generate ontologies. The ontologies created will be used to support the Scholarly Semantic Web, that is used to share, reuse, and manage scholarly data. Quan *et al*. (2006) propose a "Fuzzy Ontology Generation System" for automatically creating an ontology that incorporates FCA and fuzzy logic. This approach is later utilized by the authors [32] to construct an ontology that might be used in "web-based help-desk applications." The authors of [11] described an ontology-based retrieval strategy that allows for data organizing and visualization while also offering a user-friendly navigation mechanism. To obtain conceptual frameworks from datasets and build a hierarchal structure representation of extracted information, it employs a fuzzy extension of Formal Concept Analysis theory. This approach contributes significantly to knowledge handling. It offers hierarchy exploration and query processing after performing knowledge extraction and structuring as well as ontology-driven discovery. The outcomes of the implementation are concentrated on hierarchical facet-based navigation.

Many papers have recommended combining fuzzy logic with formal concept analysis for information retrieval. In a citation database-based document retrieval system., the authors employed FCA with fuzzy features for conceptual grouping, according to their work in [33]. Using fuzzy logic and formal concept analysis, a fuzzy concept lattice is constructed on which "a fuzzy conceptual clustering approach" is conducted. the process of getting documentation will thereafter be accomplished through the use of fuzzy queries. The authors established a methodology for developing an ontology using formal concept analysis and fuzzy features in [34]. The initial collection of documents is broken into smaller groups of similar texts using the "Growing Hierarchical Self Organizing Map clustering method." "Agglomerative clustering" is used to combine the models into a hierarchy of concept lattices. To deal with empty responses for the queries based fuzzy, the authors employed formal concept analysis with fuzzy features, according to the work in [35]. Fuzzy querying processing based on Galois lattices helps discovering reasons for empty results by displaying the subqueries that are accountable for the mistake. In [36] proposed a query expansion technique based on FCA and fuzzy attributes.

Several researchers have recently demonstrated the use of formal concept analysis in reliability engineering. In [37], the authors' goal is to present the fundamentals of FCA and how it can be applied to reliability engineering problems in their paper. To accomplish this, four examples in reliability engineering were chosen for analysis from the literature as well as the authors' personal experience. The first example explains the FCA approach based on cut-sets in network-modeled systems. The second example analyzes which protection strategy could be used to prevent various types of attack scenarios in a given network using notions inferred from knowledge space theory. The last two examples show how binary formal contexts can be extended to analyse: i) failure events caused by different reasons (granularity levels); and ii) the significance of nodes in an electric power system based on several measures of significance (attributes with multiple values).

Another research direction to use FCA in crime prediction. In this paper [38] the authors provided a brief background on crime pattern analysis as well as available methods for resolving it. Simultaneously, some of the intriguing methods are empirically analyzed based on various parameters in order to understand their appropriate applicability. They also concentrated on the uncertainty analysis that exists in crime data sets with fuzzy attributes.

5. CURRENT ISSUES AND RESEARCH DIRECTIONS

FCA is a useful formalism for representing, extracting, and analyzing whatever information system, but it has a few issues that need to be settled. In general, contexts are large, complex, and contain a huge amount of redundant information. As a result, one of the main issues identified in practical FCA applications is that the computational cost of processing the information system with FCA is high and visualizing the lattice structure is difficult [39]. Because of FCA's scalability, this complexity issue arises. Considering that counting the formal concepts in the input context is #P-complete [40], and that the number of formal concepts in the input context might be exponential, all concepts can be constructed with polynomial latency. The sizes of implication bases, even the smallest implication base, can be exponential, with the size of the stem base being #P-hard [41] to calculate.

A major challenge for FCA practical applications is the visualization of formal concepts in a hierarchy structure in the final outcome (concept lattice structure). One of the main issues with this technique is how large the concept lattice is when it is formed from a large formal context. The vast context concept lattice becomes difficult and unworkable. As a result, managing a large formal context and reducing the size of the concept lattice are highlighted as practical challenges in formal concept analysis applications. In general, procuring a concept lattice of sufficient complexity and size is one of the most fundamental challenges in formal concept analysis [39].

The literature describes a variety of techniques for controlling the complexity and size of formal contexts, formal concepts, concept lattices, and implications. To enhancing FCA scalability there are popular research techniques include iceberg concept lattices [10], matrix decompositions [42], conceptual scaling for many-valued contexts [43], the reduction of the concept lattices based on rough set theory [44], and other. In [39], the authors divided concept lattice reduction techniques into three categories. The first category of reduction techniques removes redundant information from the context, that means an object $g \in G, m \in M$ (set of attributes) or incidence $i \in I$ (I is a binary relation ($I \subseteq G \times M$)) can be considered as redundant knowledge in the formal context if removing or transforming it results in a lattice isomorphic to the orginal. The techniques for removing redundant information aim to create a concept lattice that is isomorphic to the original. The authors in their paper [45] used the same teqnique of reduction on fuzzy formal context. This category of techniques is useful when there is a lot of redundant knowledge in the formal context.

The second category of the reduction techniques is simplification techniques. The concept lattice contains all relationships between concepts, including those between concepts that are very similar. For instance, the corresponding link is shown using formal concepts that differ only by a single characteristic. The lattice can be made simpler by omitting the property that separates these concepts in these situations if it is no longer relevant (as judged by some conditions). in order to emphasize only the important knowledge. This category of techniques is useful for identifying key aspects in formal context or concept lattices.

The third category of the reductions techniques is selection techniques. Several concepts, especially in a big concept lattice, may be deemed irrelevant in a given application. The "relevance" of a concept could be related to its cardinality, intention or extension, the relationship between some attributes, and so on. Selection techniques are those that select objects, attributes or concepts based on some relevance criterion. The authors [46] made a significant contribution in this direction by connecting frequent items and formal concepts. The terms "support" and "frequent sets" are discribed as: Let $B \subseteq M$, where M is a set of attributes and $Sup(B, G)$, is the counts of objects in G that contains all the attributes of$B$. We can say that a set of attributes $B \subseteq M$ is frequent iff $Sup(B, G) \geq minSup$(minimal support previously set). Iceberg concept lattices are concept lattices created by limiting the item sets to those that are frequently used. In this instance, just the most common formal concepts are employed, leading to a partial lattice. This occurs as a result of the support for the intent being a diminishing function. In other words, the gevin a two formal concept $(A_1, B_1), (A_2, B_2)$, where $(A_1, B_1) \leq (A_2, B_2)$, sup $(B_1, G) \leq$ sup (B_2, G), where G is the sets of objects [10] describes how to create iceberg concept lattices using the Titanic method. The authors also show how to use these lattices for a number of tasks, such as large-scale database analysis, mining association rule extraction, implications extraction, and implications visualization. Following that, we summarize some of the advances in the literature on scalability issues in Table 1 (see in appendix) and briefly describe each work's contribution based on the categories that we mentioned.

6. CONCLUSION

In this work we presented an overview on the foundations of fuzzy formal concept analysis and its applications. In the literature, formal concept analysis with fuzzy attributes has gotten a lot of attention. The main focus of the researchers was on developing methods for extended FCA in a fuzzy environment like crisply generated, fuzzy concepts, generalized concept lattice, one-sided fuzzy concept lattice, and employing fuzzy formal concept analysis in domains such as reliability engineering, crime prediction, KDD, IR, and ontology engineering, An major challenge for FCA practical applications is the display of formal concepts in hierarchy structure in the concept lattice structure. One of the major issues in this process is the the large structure of the concept lattice (big line diagram) constructed from a large formal context. The concept lattice constructed from the big context becoming difficult and unworkable. Therefore, it is emphasized as a real difficulty in FCA applications to deal with a big formal context and minimize the size of the concept lattice. To deal with such problems several techniques to control the complexity and size of a concept lattice have been presented in the literature. In this work we have reviewed some of the recent articles on the reduction techniques trying to summarize the contributions.

APPENDIX

Table 1. Contributions on formal concept analysis (FCA) reduction techniques

Papers	Work's contribution	Category
[1]	By removing reducible objects and attributes, the authors were able to obtain a clarified context, and the resulting concept lattice maintains the isomorphism with the original.	Redundant information removal
[47]	The authors demonstrated how to factorize concept lattices based on concept similarity. It's also demonstrated how to speed up the computation of similarity relations. They defined and examined the similarity relations at three levels: similarity of objects and attributes, similarity of concepts, and similarity of concept lattices.	Redundant information removal
[48]	This paper investigates the granular structure of concept lattices and how it can be used to reduce knowledge in formal concept analysis. The properties of information granules are first discussed in a formal context. In the formal context, the concepts of a granular consistent set and granular reducts are introduced.	Redundant information removal
[49]	The attributes in a decision formal context have been reduced using a homomorphism consistent set from the concept lattice.	Redundant information removal
[50]	The author proposes a framework for knowledge reduction from a decision formal context that employs rule acquisition to discover a new set of non-redundant decision rules.	Redundant information removal
[51]	Ciobanu later used the reduction described in [47] to introduce a new reduction in the property-oriented and object-oriented concept lattice frameworks.	Redundant information removal
[52]	The authors in their article covered two important research topics in FCA: attribute reduction and size reduction in concept lattices. The authors present a procedure that uses an irreducible -cut concept lattice to simultaneously reduce attributes and concept lattice size.	Redundant information removal
[53]	JBOS (Junction based on object similarity) uses background knowledge to replace similar objects with representative elements that are similar to a certain degree.	Simplification
[54]	Using fuzzy k means (FKM) clustering, the size of the concept lattices was reduced. The context matrix is reduced, and quotient lattices are obtained using FKM Clustering-derived equivalence relations. Each element is associated with a set of membership levels, and each record can belong to more than one cluster.	Simplification
	The authors extended previous work on the correspondence of block relations of formal contexts and complete tolerances on concept lattices to a fuzzy setting and provided an example of how to use block relations to reduce a concept lattice.	Simplification
[55]	The authors investigated concept lattices in uncertain environments. They investigated the fuzziness in a multivalued context, which is then transformed into fuzzy formal contexts and fuzzy formal concepts. By simplifying the corresponding fuzzy concept lattice structure, they were able to reduce the number of fuzzy formal concepts.	Simplification
[56]	Based on their characteristics, fuzzy formal contexts are reduced using attribute reduction. The term "one-sided fuzzy concept" is used for the first time. The attributes are divided into three categories: core attributes, relatively important attributes, and unimportant attributes. By virtue of attribute characteristics, an attribute reduction method is presented.	Simplification
[57]	They've implemented a mechanism to reduce attributes in fuzzy FCA, considering the reduction procedure and tolerance relations introduced in RST. This new method for reducing attributes reduces the original concept lattice significantly. The most important feature of this method is that it partially preserves the structure of the original concept lattice when using this new mechanism, i.e., no new join-irreducible elements appear after the reduction procedure.	Simplification
[10]	The authors made a significant contribution by connecting frequent items and formal concepts as described in [57].	Selection
[58]	The authors introduced a type of concept lattice, which are like iceberg concept lattices. Some class restraints are built with attributes in a formal context. The authors named the resulting concept lattice alpha concept lattice. An iceberg concept lattice is formed by an unrestricted lattice, which contains only frequent formal concepts.	Selection
[59]	The choice of formal concepts in the proposed method is based on the concept of distance or similarity. In the process of selecting important concepts, the concepts of equivalence classes and object or attribute similarity are used.	Selection
[4]	In this work, each attribute is given a weight to demonstrate its relevance, and thereafter formal concepts that are relevant are chosen. Equal weights are assigned to attributes derived from multivalued attributes to facilitate the application of weights. The sum of the weights of a formal concept's attributes intention divided by the cardinality of its intention determines its importance.	Selection
[45]	The authors introduce the Titanic algorithm for generating iceberg concept lattices and demonstrate the utility of these lattices in a variety of applications, including large-scale database analysis, extraction of implications, visualization of implications and mining association rules.	Selection
[60]	For fuzzy formal concepts, the authors presented a similarity metric. In order to choose a subset of formal concepts that are related to one another, the similarity measure is utilized. This subset of formal concepts may be much smaller than the initial set of formal concepts. A measurement of similarities between formal concept extensions is defined as follows. Given two forma concepts $(A_1, B_1), (A_2, B_2)$, the similarity between the extentions $A_1$and A_2 given by $Sim(A_1, A_2)$=1- $\|A_1' \cap A_2'\|/ \|A_1' \cup A_2'\|$.|	Selection
[61]	The authors focused on using entropy to reduce the number of formal concepts in formal concept analysis with fuzzy attributes. Furthermore, at a given granulation of the entropy-based attribute intent weight, the number of fuzzy formal concepts is reduced.	Selection

REFERENCES

[1] B. Ganter and R. Wille, “Formal Concept Analysis : Mathematical Foundations Formal Concept Analysis,” *Springer*, 1997, doi: 10.1007/978-3-642-59830-2.

[2] B. A. Davey and H. A. Priestley, “Introduction to Lattices and Order,” *Introduction to Lattices and Order*, 2002, doi: 10.1017/cbo9780511809088.

[3] L. A. Zadeh, *Information and control*, vol. 8, no. 3. 1965.

[4] R. Belohlavek and J. Macko, “Selecting important concepts using weights,” *Lecture Notes in Computer Science (including*

subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), vol. 6628 LNAI, pp. 65–80, 2011, doi: 10.1007/978-3-642-20514-9_7.

[5] M. Klimushkin, S. Obiedkov, and C. Roth, "Approaches to the selection of relevant concepts in the case of noisy data," *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, vol. 5986 LNAI, pp. 255–266, 2010, doi: 10.1007/978-3-642-11928-6_18.

[6] M. D. Rice and M. Siff, "Clusters, concepts, and pseudo-metrics," *Electronic Notes in Theoretical Computer Science*, vol. 40, pp. 323–346, 2001, doi: 10.1016/S1571-0661(05)80060-X.

[7] J. Medina, "Relating attribute reduction in formal, object-oriented and property-oriented concept lattices," *Computers and Mathematics with Applications*, vol. 64, no. 6, pp. 1992–2002, 2012, doi: 10.1016/j.camwa.2012.03.087.

[8] H. Wang and W. X. Zhang, "Approaches to knowledge reduction in generalized consistent decision formal context," *Mathematical and Computer Modelling*, vol. 48, no. 11–12, pp. 1677–1684, 2008, doi: 10.1016/j.mcm.2008.06.007.

[9] R. Belohlavek and V. Vychodil, "Formal concept analysis with background knowledge: Attribute priorities," *IEEE Transactions on Systems, Man and Cybernetics Part C: Applications and Reviews*, vol. 39, no. 4, pp. 399–409, 2009, doi: 10.1109/TSMCC.2008.2012168.

[10] G. Stumme, R. Taouil, Y. Bastide, N. Pasquier, and L. Lakhal, "Computing iceberg concept lattices with TITANIC," *Data and Knowledge Engineering*, vol. 42, no. 2, pp. 189–222, 2002, doi: 10.1016/S0169-023X(02)00057-5.

[11] C. De Maio, G. Fenza, V. Loia, and S. Senatore, "Hierarchical web resources retrieval by exploiting fuzzy formal concept analysis," *Information Processing and Management*, vol. 48, no. 3, pp. 399–418, 2012, doi: 10.1016/j.ipm.2011.04.003.

[12] A. Burusco and R. Fuentes-González, "Construction of the L-fuzzy concept lattice," *Fuzzy Sets and Systems*, vol. 97, no. 1, pp. 109–114, 1998, doi: 10.1016/S0165-0114(96)00318-1.

[13] R. Bělohlávek, "Lattices of fixed points of fuzzy Galois connections," *Mathematical Logic Quarterly: Mathematical Logic Quarterly*, vol. 47, no. 1, pp. 111–116, 2001, doi: 10.1002/1521-3870.

[14] R. Bělohlávek, "Fuzzy Galois connections," *Mathematical Logic Quarterly*, vol. 45, no. 4, pp. 497–504, 1999, doi: 10.1002/malq.19990450408.

[15] C. J. H. Mann, "Fuzzy Relational Systems: Foundations and Principles," *Kybernetes*, vol. 32, no. 9/10, 2003, doi: 10.1108/k.2003.06732iae.005.

[16] a. B. Juandeaburre and R. Fuentes-González, "The study of the L-fuzzy concept lattice," *Mathware and Soft Computing*, vol. 3, no. 3, pp. 209–218, 1994.

[17] A. Burusco and R. Fuentes-González, "Concept lattices defined from implication operators," *Fuzzy Sets and Systems*, vol. 114, no. 3, pp. 431–436, 2000, doi: 10.1016/S0165-0114(98)00182-1.

[18] A. Burusco and R. Fuentes-Gonzalez, "Contexts with multiple weighted values," *International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems*, vol. 9, no. 3, pp. 355–368, 2001, doi: 10.1142/S0218488501000843.

[19] R. Belohlavek, "Lattices generated by binary fuzzy relations (extended abstract)," *Abstracts of the Fourth International Conference on Fuzzy Sets Theory and Its Applications*, vol. 11, p. 11, 1998.

[20] S. Pollandt, "Fuzzy-Kontexte," *Fuzzy-Begriffe*, pp. 21–49, 1997, doi: 10.1007/978-3-642-60460-7_3.

[21] P. Hájek, "Metamathematics of fuzzy logic," *Springer Dordrecht*, 1998, doi: 10.1007/978-94-011-5300-3.

[22] G. Georgescu and A. Popescu, "Concept lattices and similarity in non-commutative fuzzy logic," *Fundamenta Informaticae*, vol. 53, no. 1, pp. 23–54, 2002.

[23] S. Ben Yahia and A. Jaoua, "Discovering Knowledge from Fuzzy Concept Lattice," pp. 167–190, 2001, doi: 10.1007/978-3-7908-1825-3_7.

[24] S. Krajči, "Cluster based efficient generation of fuzzy concepts," *Neural Network World*, vol. 13, no. 5, pp. 521–530, 2003.

[25] R. Bělohlávek, V. Sklenář, and J. Zacpal, "Crisply generated fuzzy concepts," *Lecture Notes in Artificial Intelligence (Subseries of Lecture Notes in Computer Science)*, vol. 3403, pp. 269–284, 2005, doi: 10.1007/978-3-540-32262-7_19.

[26] S. Krajči, "A generalized concept lattice," *Logic Journal of the IGPL*, vol. 13, no. 5, pp. 543–550, 2005, doi: 10.1093/jigpal/jzi045.

[27] S. Krajči, "The basic theorem on generalized concept lattice," *CEUR Workshop Proceedings*, vol. 110, pp. 25–33, 2004.

[28] V. Sklenář, J. Zacpal, and E. Sigmund, "Evaluation of IPAQ questionnaire by FCA," *CEUR Workshop Proceedings*, vol. 162, pp. 60–69, 2005.

[29] A. Bertaux, F. Le Ber, A. Braud, and M. Trémolières, "Identifying ecological traits: A concrete FCA-based approach," *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, vol. 5548 LNAI, pp. 224–236, 2009, doi: 10.1007/978-3-642-01815-2_17.

[30] G. Fenza and S. Senatore, "Friendly web services selection exploiting fuzzy formal concept analysis," *Soft Computing*, vol. 14, no. 8, pp. 811–819, 2010, doi: 10.1007/s00500-009-0469-2.

[31] T. T. Quan, S. C. Hui, A. C. M. Fong, and T. H. Cao, "Automatic generation of ontology for scholarly semantic web," *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, vol. 3298, pp. 726–740, 2004, doi: 10.1007/978-3-540-30475-3_50.

[32] T. T. Quan, S. C. Hui, and A. C. M. Fong, "Automatic fuzzy ontology generation for semantic help-desk support," *IEEE Transactions on Industrial Informatics*, vol. 2, no. 3, pp. 155–163, 2006, doi: 10.1109/TII.2006.873363.

[33] T. T. Quan, S. C. Hui, and T. H. Cao, "A fuzzy FCA-based approach for citation-based document retrieval," *2004 IEEE Conference on Cybernetics and Intelligent Systems*, pp. 577–582, 2004, doi: 10.1109/iccis.2004.1460480.

[34] P. Butka, M. Sarnovsky, and P. Bednar, "One approach to combination of FCA-based local conceptual models for text analysis - Grid-based approach," *SAMI 2008 6th International Symposium on Applied Machine Intelligence and Informatics - Proceedings*, pp. 131–135, 2008, doi: 10.1109/SAMI.2008.4469150.

[35] H. Chettaoui, N. Hachani, M. A. Ben Hassine, and H. Ounelli, "Using FCA to answer fuzzy queries in cooperative systems," *Proceedings - 5th International Conference on Fuzzy Systems and Knowledge Discovery, FSKD 2008*, vol. 3, pp. 14–20, 2008, doi: 10.1109/FSKD.2008.327.

[36] B. Zhang, H. M. Li, Y. J. Du, and Y. T. Wang, "Query expansion based on topics," *Proceedings - 5th International Conference on Fuzzy Systems and Knowledge Discovery, FSKD 2008*, vol. 2, pp. 610–614, 2008, doi: 10.1109/FSKD.2008.464.

[37] C. M. Rocco, E. Hernandez-Perdomo, and J. Mun, "Introduction to formal concept analysis and its applications in reliability engineering," *Reliability Engineering and System Safety*, vol. 202, 2020, doi: 10.1016/j.ress.2020.107002.

[38] P. Kapoor, P. K. Singh, and A. K. Cherukuri, "Crime data set analysis using formal concept analysis (fca): A survey," *Lecture Notes in Electrical Engineering*, vol. 612, pp. 15–31, 2020, doi: 10.1007/978-981-15-0372-6_2.

[39] S. M. Dias and N. J. Vieira, "Concept lattices reduction: Definition, analysis and classification," *Expert Systems with Applications*, vol. 42, no. 20, pp. 7084–7097, 2015, doi: 10.1016/j.eswa.2015.04.044.

[40] S. O. Kuznetsov, "On Computing the Size of a Lattice and Related Decision Problems," *Order*, vol. 18, no. 4, pp. 313–321, 2001,

doi: 10.1023/A:1013970520933.
[41] S. O. Kuznetsov, "On the intractability of computing the duquenne-guigues base," *Journal of Universal Computer Science*, vol. 10, no. 8, pp. 927–933, 2004.
[42] V. Snasel, P. Gajdos, H. M. D. Abdulla, and M. Polovincak, "Using matrix decompositions in formal concept analysis," *CEUR Workshop Proceedings*, vol. 252, pp. 121–128, 2007.
[43] N. Messai, M. D. Devignes, A. Napoli, and M. Smail-Tabbone, "Many-valued concept lattices for conceptual clustering and information retrieval," *Frontiers in Artificial Intelligence and Applications*, vol. 178, pp. 127–131, 2008, doi: 10.3233/978-1-58603-891-5-127.
[44] M. Liu, M. Shao, W. Zhang, and C. Wu, "Reduction method for concept lattices based on rough set theory and its application," *Computers and Mathematics with Applications*, vol. 53, no. 9, pp. 1390–1410, 2007, doi: 10.1016/j.camwa.2006.03.040.
[45] D. Pei, M. Z. Li, and J. S. Mi, "Attribute reduction in fuzzy decision formal contexts," *Proceedings - International Conference on Machine Learning and Cybernetics*, vol. 1, pp. 204–208, 2011, doi: 10.1109/ICMLC.2011.6016665.
[46] N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, "Discovering frequent closed itemsets for association rules," *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, vol. 1540, pp. 398–416, 1998, doi: 10.1007/3-540-49257-7_25.
[47] R. Bělohlávek, "Similarity relations in concept lattices," *Journal of Logic and Computation*, vol. 10, no. 6, pp. 823–845, 2000, doi: 10.1093/logcom/10.6.823.
[48] W. Z. Wu, Y. Leung, and J. S. Mi, "Granular computing and knowledge reduction in formal contexts," *IEEE Transactions on Knowledge and Data Engineering*, vol. 21, no. 10, pp. 1461–1474, 2009, doi: 10.1109/TKDE.2008.223.
[49] D. Pei and J. S. Mi, "Attribute reduction in decision formal context based on homomorphism," *International Journal of Machine Learning and Cybernetics*, vol. 2, no. 4, pp. 289–293, 2011, doi: 10.1007/s13042-011-0034-z.
[50] J. Li, C. Mei, C. A. Kumar, and X. Zhang, "On rule acquisition in decision formal contexts," *International Journal of Machine Learning and Cybernetics*, vol. 4, no. 6, pp. 721–731, 2013, doi: 10.1007/s13042-013-0150-z.
[51] G. Ciobanu and C. Vəideanu, "Similarity relations in fuzzy attribute-oriented concept lattices," *Fuzzy Sets and Systems*, vol. 275, pp. 88–109, 2015, doi: 10.1016/j.fss.2014.12.011.
[52] M. E. Cornejo, J. Medina, and E. Ramírez-Poussa, "Attribute and size reduction mechanisms in multi-adjoint concept lattices," *Journal of Computational and Applied Mathematics*, vol. 318, pp. 388–402, 2017, doi: 10.1016/j.cam.2016.07.012.
[53] S. M. Dias and N. J. Vieira, "Applying the JBOS reduction method for relevant knowledge extraction," *Expert Systems with Applications*, vol. 40, no. 5, pp. 1880–1887, 2013, doi: 10.1016/j.eswa.2012.10.010.
[54] C. Aswani Kumar and S. Srinivas, "Concept lattice reduction using fuzzy K-Means clustering," *Expert Systems with Applications*, vol. 37, no. 3, pp. 2696–2704, 2010, doi: 10.1016/j.eswa.2009.09.026.
[55] P. K. Singh, C. Aswani Kumar, and J. Li, "Knowledge representation using interval-valued fuzzy formal concept lattice," *Soft Computing*, vol. 20, no. 4, pp. 1485–1502, 2016, doi: 10.1007/s00500-015-1600-1.
[56] J. Li, C. Huang, J. Qi, Y. Qian, and W. Liu, "Three-way cognitive concept learning via multi-granularity," *Information Sciences*, vol. 378, pp. 244–263, 2017, doi: 10.1016/j.ins.2016.04.051.
[57] M. J. Benítez-Caballero, J. Medina, E. Ramírez-Poussa, and D. Ślęzak, "Rough-set-driven approach for attribute reduction in fuzzy formal concept analysis," *Fuzzy Sets and Systems*, vol. 391, pp. 117–138, 2020, doi: 10.1016/j.fss.2019.11.009.
[58] H. Soldano, V. Ventos, M. Champesme, and D. Forge, "Incremental construction of alpha lattices and association rules," *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, vol. 6277 LNAI, no. PART 2, pp. 351–360, 2010, doi: 10.1007/978-3-642-15390-7_36.
[59] V. Codocedo, C. Taramasco, and H. Astudillo, "Cheating to achieve Formal Concept analysis over a large formal context," *CEUR Workshop Proceedings*, vol. 959, pp. 349–362, 2011.
[60] A. Formica, "Similarity reasoning for the semantic web based on fuzzy concept lattices: An informal approach," *Information Systems Frontiers*, vol. 15, no. 3, pp. 511–520, 2013, doi: 10.1007/s10796-011-9340-y.
[61] P. K. Singh, A. K. Cherukuri, and J. Li, "Concepts reduction in formal concept analysis with fuzzy setting using Shannon entropy," *International Journal of Machine Learning and Cybernetics*, vol. 8, no. 1, pp. 179–189, 2017, doi: 10.1007/s13042-014-0313-6.

BIOGRAPHIES OF AUTHORS

Alwersh Mohammed received the B.Sc. degree in Computer Science in 2008 from the University of Al-Qaddissiyah, Iraq, and in 2014 the M.Sc. degree in Information Technology from Belarusian state university of informatics and radioelectronics (BSUIR), Minsk-Belarus. He is a PhD student at the University of Miskolc, Miskolc Hungary, since 2020. The Topic title is "Attribute Set Reduction of Categorical Attributes for Machine Learning and FCA Methods". He can be contacted at email: alwersh.mohammed.ali.daash@student.uni-miskolc.hu.

Prof. Dr. László Kovács Head of the Department of the Institute of Informatics of the University of Miskolc. His main research interests are optimization of neural network-based prediction methods, theory of concept lattices, NLP and automatic question generation procedures, applications of ontology in knowledge engineering. He is an author of more than 90 journal publications in the related research fields. He is an active member of the editorial board at 7 international journals. He can be contacted at email: kovacs@iit.uni-miskolc.hu.